5-1 Midsegments of Triangles

Objective To use properties of midsegments to solve problems

SOLVE IT!

Getting Ready!

Cut out a triangle of any shape. Label its largest angle C, and the other angles A and B. Fold A onto C to find the midpoint of $\overline{AC}$. Do the same for $\overline{BC}$. Label the midpoints L and N, and then draw $\overline{LN}$.
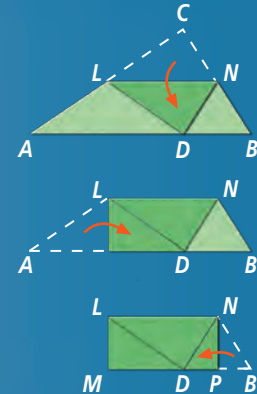
Fold the triangle on $\overline{LN}$ as shown.

Fold A to D and fold B to D.

Label the vertices M and P as shown. What is the relationship between MP and AB? How do you know? What conjecture can you make about the relationship between LN and AB?

Check with your classmates. Do they get the same results?

MATHEMATICAL PRACTICES

Lesson Vocabulary
• midsegment of a triangle

In the Solve It, $\overline{LN}$ is a midsegment of $\triangle ABC$. A **midsegment of a triangle** is a segment connecting the midpoints of two sides of the triangle.

Essential Understanding There are two special relationships between a midsegment of a triangle and the third side of the triangle.
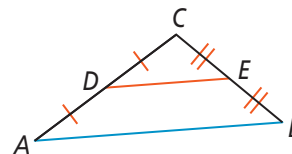
take note

Theorem 5-1 Triangle Midsegment Theorem

| **Theorem**	**If . . .**	**Then . . .**
If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and is half as long.	D is the midpoint of $\overline{CA}$ and E is the midpoint of $\overline{CB}$	$\overline{DE} \parallel \overline{AB}$ and $DE = \frac{1}{2}AB$

You will prove Theorem 5-1 in Lesson 6-9.

Here's Why It Works You can verify that the Triangle Midsegment Theorem works for a particular triangle. Use the following steps to show that $\overline{DE} \parallel \overline{AB}$ and that $DE = \frac{1}{2}AB$ for a triangle with vertices at $A(4, 6)$, $B(6, 0)$, and $C(0, 0)$, where D and E are the midpoints of $\overline{CA}$ and $\overline{CB}$.
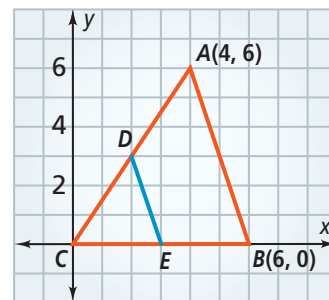
Step 1 Use the Midpoint Formula, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$, to find the coordinates of D and E.

The midpoint of $\overline{CA}$ is $D\left(\frac{0+4}{2}, \frac{0+6}{2} \right) = D(2, 3)$.

The midpoint of $\overline{CB}$ is $E\left(\frac{0+6}{2}, \frac{0+0}{2} \right) = E(3, 0)$.

Step 2 To show that the midsegment $\overline{DE}$ is parallel to the side $\overline{AB}$, find the slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$, of each segment.

$$\text{slope of } \overline{DE} = \frac{0 - 3}{3 - 2} \qquad \text{slope of } \overline{AB} = \frac{0 - 6}{6 - 4}$$

$$= \frac{-3}{1} \qquad\qquad\qquad = \frac{-6}{2}$$

$$= -3 \qquad\qquad\qquad\quad = -3$$

Step 3 To show $DE = \frac{1}{2}AB$, use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find DE and AB.

$$DE = \sqrt{(3 - 2)^2 + (0 - 3)^2} \qquad AB = \sqrt{(6 - 4)^2 + (0 - 6)^2}$$

$$= \sqrt{1 + 9} \qquad\qquad\qquad\quad = \sqrt{4 + 36}$$

$$= \sqrt{10} \qquad\qquad\qquad\qquad = \sqrt{40}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad = 2\sqrt{10}$$

Since $\sqrt{10} = \frac{1}{2}(2\sqrt{10})$, you know that $DE = \frac{1}{2}AB$.


Problem 1 **Identifying Parallel Segments**

What are the three pairs of parallel segments in $\triangle DEF$?

$\overline{RS}$, $\overline{ST}$, and $\overline{TR}$ are the midsegments of $\triangle DEF$. By the Triangle Midsegment Theorem, $\overline{RS} \parallel \overline{DF}$, $\overline{ST} \parallel \overline{ED}$, and $\overline{TR} \parallel \overline{FE}$.
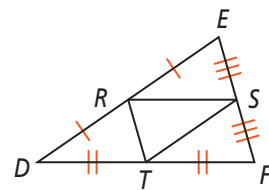
Got It? **1. a.** In $\triangle XYZ$, A is the midpoint of $\overline{XY}$, B is the midpoint of $\overline{YZ}$, and C is the midpoint of $\overline{ZX}$. What are the three pairs of parallel segments?

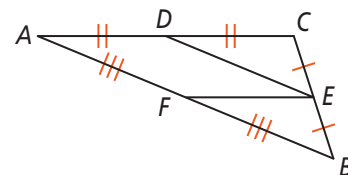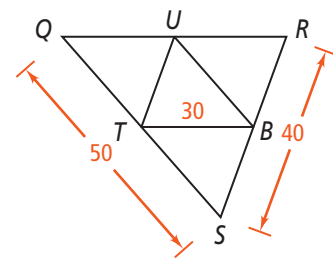**b. Reasoning** What is $m\angle VUO$ in the figure at the right? Explain your reasoning.
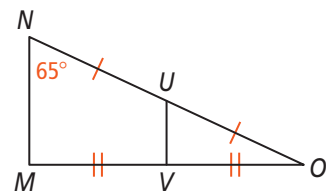
Problem 2 Finding Lengths

In $\triangle QRS$, T, U, and B are midpoints. What are the lengths of $\overline{TU}$, $\overline{UB}$, and $\overline{QR}$?

Use the relationship

length of a midsegment $= \frac{1}{2}$ (length of the third side)

to write an equation about the length of each midsegment.

$TU = \frac{1}{2}SR$	$UB = \frac{1}{2}QS$	$TB = \frac{1}{2}QR$
$= \frac{1}{2}(40)$	$= \frac{1}{2}(50)$	$30 = \frac{1}{2}QR$
$= 20$	$= 25$	$60 = QR$

✅ **Got It?** **2.** In the figure at the right, $AD = 6$ and $DE = 7.5$. What are the lengths of $\overline{DC}$, $\overline{AC}$, $\overline{EF}$, and $\overline{AB}$?

You can use the Triangle Midsegment Theorem to find lengths of segments that might be difficult to measure directly.

© **Problem 3** Using a Midsegment of a Triangle STEM

Environmental Science A geologist wants to determine the distance, *AB*, across a sinkhole. Choosing a point *E* outside the sinkhole, she finds the distances *AE* and *BE*. She locates the midpoints *C* and *D* of $\overline{AE}$ and $\overline{BE}$ and then measures $\overline{CD}$. What is the distance across the sinkhole?
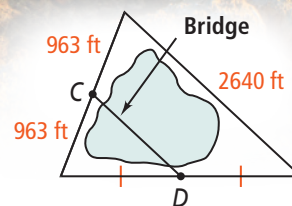
$\overline{CD}$ is a midsegment of $\triangle AEB$.

$CD = \frac{1}{2}AB$	$\triangle$ Midsegment Thm.
$46 = \frac{1}{2}AB$	Substitute 46 for *CD*.
$92 = AB$	Multiply each side by 2.

The distance across the sinkhole is 92 ft.
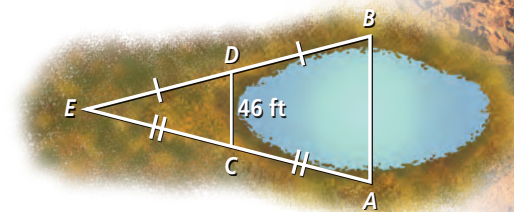
✅ **Got It?** **3.** $\overline{CD}$ is a bridge being built over a lake, as shown in the figure at the right. What is the length of the bridge?
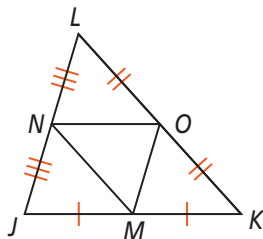
✓ Lesson Check

Do you know HOW?

Use the figure at the right for Exercises 1–3.

1. Which segment is parallel to $\overline{JK}$?

2. If $LK = 46$, what is NM?

3. If $JK = 5x + 20$ and $NO = 20$, what is the value of x?

Do you UNDERSTAND? Ⓒ MATHEMATICAL PRACTICES

Ⓒ 4. **Vocabulary** How does the term *midsegment* describe the segments discussed in this lesson?

Ⓒ 5. **Reasoning** If two noncollinear segments in the coordinate plane have slope 3, what can you conclude?

Ⓒ 6. **Error Analysis** A student sees this figure and concludes that $\overline{PL} \parallel \overline{NO}$. What is the error in the student's reasoning?
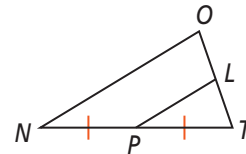
Practice and Problem-Solving Exercises   Ⓒ MATHEMATICAL PRACTICES
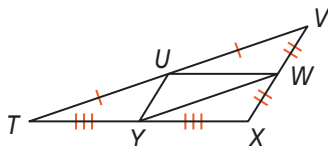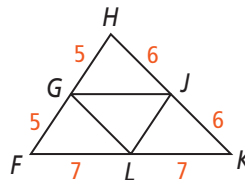
Ⓐ Practice   Identify three pairs of parallel segments in each diagram.   ◀ See Problem 1.

7.

8.

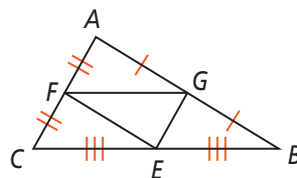Name the segment that is parallel to the given segment.

9. $\overline{AB}$ 10. $\overline{BC}$

11. $\overline{EF}$ 12. $\overline{CA}$

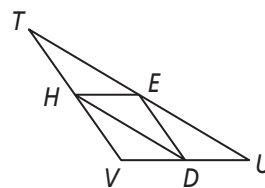13. $\overline{GE}$ 14. $\overline{FG}$

Points E, D, and H are the midpoints of the sides of $\triangle TUV$. $UV = 80$, $TV = 100$, and $HD = 80$.   ◀ See Problem 2.
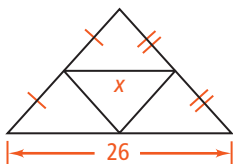
15. Find HE.   16. Find ED.

17. Find TU.   18. Find TE.

Algebra Find the value of x.

19.

20.
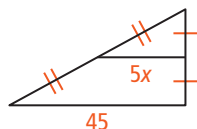
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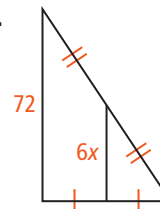
Algebra Find the value of *x*.

22.

17

x − 4

23.

38

x + 2

24.

5*x* − 4

8
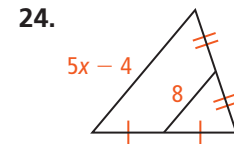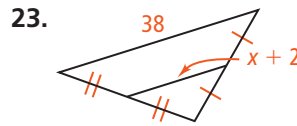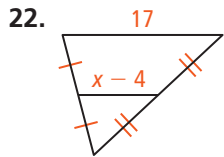
**25. Surveying** A surveyor needs to measure the distance *PQ* across the lake. Beginning at point *S*, she locates the midpoints of $\overline{SQ}$ and $\overline{SP}$ at *M* and *N*. She then measures $\overline{NM}$. What is *PQ*?

P *N* *S*

Q *M* 78 m

B Apply
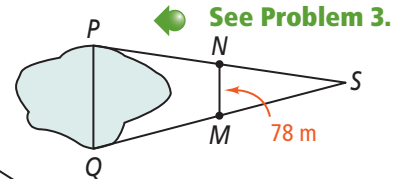
26. Kayaking You want to paddle your kayak across a lake. To determine how far you must paddle, you pace out a triangle, counting the number of strides, as shown.
   a. If your strides average 3.5 ft, what is the length of the longest side of the triangle?
   b. What distance must you paddle across the lake?

80 150

80 150

250
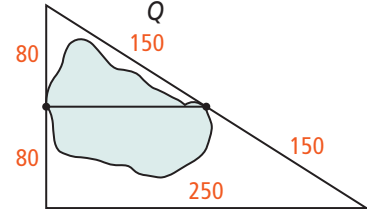
27. Architecture The triangular face of the Rock and Roll Hall of Fame in Cleveland, Ohio, is isosceles. The length of the base is 229 ft 6 in. Each leg is divided into four congruents parts by the red segments. What is the length of the white segment? Explain your reasoning.
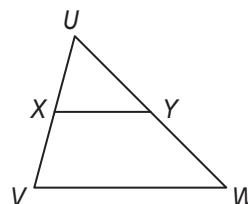

ROCK AND ROLL HALL OF FAME AND MUSEUM
ONE KEY PLAZA

28. Think About a Plan Draw △*ABC*. Construct another triangle so that the three sides of △*ABC* are the midsegments of the new triangle.
   • Can you visualize or sketch the final figure?
   • Which segments in your final construction will be parallel?

29. Writing In the figure at the right, $m\angle QST = 40$. What is $m\angle QPR$? Explain how you know.
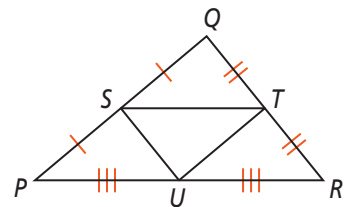
30. Coordinate Geometry The coordinates of the vertices of a triangle are *E*(1, 2), *F*(5, 6), and *G*(3, −2).
   a. Find the coordinates of *H*, the midpoint of $\overline{EG}$, and *J*, the midpoint of $\overline{FG}$.
   b. Show that $\overline{HJ} \parallel \overline{EF}$.
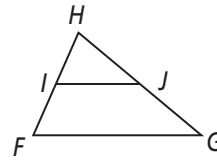   **c.** Show that $HJ = \frac{1}{2}EF$.

Q

S *T*

P *U* *R*

X is the midpoint of $\overline{UV}$. *Y* is the midpoint of $\overline{UW}$.

31. If $m\angle UXY = 60$, find $m\angle V$.

32. If $m\angle W = 45$, find $m\angle UYX$.

33. If $XY = 50$, find *VW*.

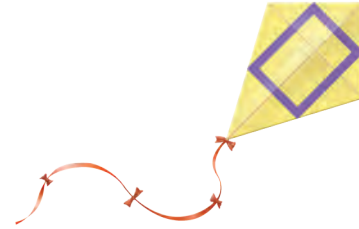**34.** If $VW = 110$, find *XY*.

U

X *Y*

V *W*

$\overline{IJ}$ is a midsegment of $\triangle FGH$. $IJ = 7$, $FH = 10$, and $GH = 13$. Find the perimeter of each triangle.
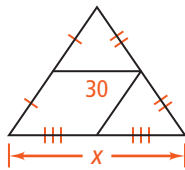
35. $\triangle IJH$

36. $\triangle FGH$

37. Kite Design You design a kite to look like the one at the right. Its diagonals measure 64 cm and 90 cm. You plan to use ribbon, represented by the purple rectangle, to connect the midpoints of its sides. How much ribbon do you need?

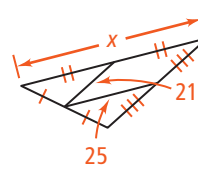   Ⓐ 77 cm   Ⓒ 154 cm

    Ⓑ 122 cm   Ⓓ 308 cm

Algebra Find the value of each variable.
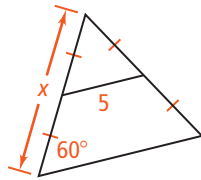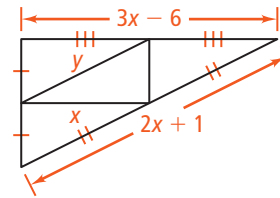
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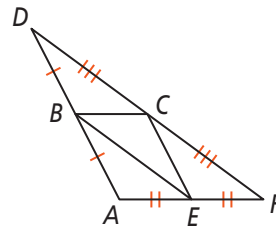
39.

40.

41.

Use the figure at the right for Exercises 42–44.

42. $DF = 24$, $BC = 6$, and $DB = 8$. Find the perimeter of $\triangle ADF$.

43. Algebra If $BE = 2x + 6$ and $DF = 5x + 9$, find DF.

44. Algebra If $EC = 3x - 1$ and $AD = 5x + 7$, find EC.

Ⓒ **45. Open-Ended** Explain how you could use the Triangle Midsegment Theorem as the basis for this construction: Draw $\overline{CD}$. Draw point A not on $\overline{CD}$. Construct $\overline{AB}$ so that $\overline{AB} \parallel \overline{CD}$ and $AB = \frac{1}{2}CD$.

Ⓒ **Challenge** Ⓒ **46. Reasoning** In the diagram at the right, K, L, and M are the midpoints of the sides of $\triangle ABC$. The vertices of the three small purple triangles are the midpoints of the sides of $\triangle KBL$, $\triangle AKM$, and $\triangle MLC$. The perimeter of $\triangle ABC$ is 24 cm. What is the perimeter of the shaded region?
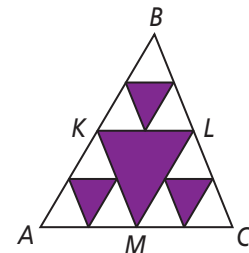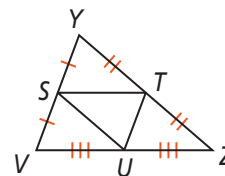
47. Coordinate Geometry In $\triangle GHJ$, $K(2, 3)$ is the midpoint of $\overline{GH}$, $L(4, 1)$ is the midpoint of $\overline{HJ}$, and $M(6, 2)$ is the midpoint of $\overline{GJ}$. Find the coordinates of G, H, and J.

48. Complete the Prove statement and then write a proof.

Proof **Given:** In $\triangle VYZ$, S, T, and U are midpoints.
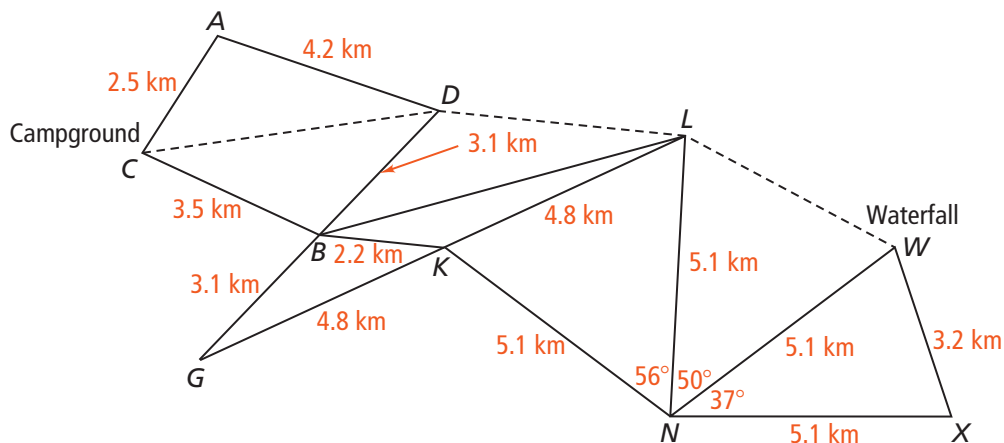
Prove: $\triangle YST \cong \triangle TUZ \cong \triangle SVU \cong$ <u>?</u>

Apply What You've Learned

Look at the trail map from page 283, shown again below.

Select all of the following that are true. Explain your reasoning.

A. In $\triangle DGL$, $\overline{BL}$ is a midsegment.

B. In $\triangle DGL$, $\overline{BK}$ is a midsegment.

C. $\overline{BK}$ is parallel to $\overline{DL}$.

D. $\overline{BL}$ is parallel to $\overline{GK}$.

E. The length of $\overline{BK}$ is half the length of $\overline{BL}$.

F. The length of $\overline{DL}$ is twice the length of $\overline{BK}$.

G. $\overline{DL}$ is the shortest side of $\triangle DGL$.